Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences

Tome 66, No 5, 2013

MATHEMATIQUES
Sous-ensembles flous

FUZZY LOGIC Q-MEASURE MODEL FOR MANAGING FINANCIAL INVESTMENTS

Penka Georgieva, Ivan Popchev*

(*Submitted on December 19, 2012*)

Abstract

In this paper a fuzzy model for supporting the process of financial investing is proposed. The model uses the tools of soft computing as it is based on fuzzy logic and comprises the two major economic concepts: return and risk, combined with one additional characteristic – the q-ratio. In the proposed model there are no assumptions for existence of probability distributions of asset returns. The outcome of the model is a Q-measure which is used for managing an individual asset as well as for portfolio management.

Key words: fuzzy system, investment, portfolio management
2010 Mathematics Subject Classification: 68T37, 03E72

1. Introduction. The proposed model emanates from a simple concept: every investor has one goal which is achieving maximum return at minimum risk. Therefore, the key point in the process of managing financial investments is finding a reliable estimator for changes in asset prices. Most financial models are built on the assumption that asset returns have some type of probability distribution. For instance, the Random Walk Hypothesis is based on the assumption that the increments of returns are independently and identically distributed. Some variations of this hypothesis are based on existence of normal distribution of increments of returns. In the Capital Asset Price Model there is an analogical assumption – the returns should be independently and identically distributed through time and jointly multivariate normally distributed. The Arbitrage Price theory is based on the joint normality assumption for the returns. In general, the classical models are based on some form of probabilities [3,8–10,12,13,15,16]. However, empirical tests conducted on data from Bulgarian Stock Exchange prove the opposite [6].

Fuzzy logic provides adequate tools for dealing with a large amount of data (such as time series of asset prices) and sometimes vague and imprecise information (as is economic information) [1]. What is more, in fuzzy modelling there are no requirements for existence of probability distributions [2,5,23].

The proposed model implements the annual log return as an estimator of asset return, the standard deviation of log returns as an estimator of investment risk and their quotient q-ratio as an indicator for stability in return changes.

2. Description of fuzzy logic Q-measure model (FLQM model). The goal of decision making is to find an optimal solution for a situation where a number of possible solutions exists. Bellman and Zadeh [23] proposed a fuzzy model for decision-making in which objectives and goals are described as fuzzy sets and the solution is a suitable aggregation of these sets. There are various algorithms for building a fuzzy system [11, 17–20].

2.1. Input variables. The calculations of the crisp values of the input variables: annual return, risk and q-ratio are derived in [7]. These crisp values are fuzzified with the predefined linguistic variables (LVs). The output variable is one – the Q-measure of an asset. In the description of the LVs, definitions and notations in [22] are followed.

The names of LVs are $X_1 \triangleq$ Return, $X_2 \triangleq$ Risk, $X_3 \triangleq$ q-ratio, $Y \triangleq$ Q-measure, The term-sets of LVs are $T(X_1) = \{X_{1j}\}$, $T(X_2) = \{X_{2j}\}$, $T(X_3) = \{X_{3k}\}$, $T(Y) = \{Y_j\}$, for $j = 1, \dots, 5$; $k = 1, 2, 3$ and

$$X_{ij} \triangleq \begin{pmatrix} \text{Very low} & i=1,2 & j=1 \\ \text{Low} & i=1,2 & j=2 \\ \text{Neutral} & i=1,2 & j=3 \\ \text{High} & i=1,2 & j=4 \\ \text{Very high} & i=1,2 & j=5 \\ \text{Small} & i=3 & j=1 \\ \text{Neutral} & i=3 & j=2 \\ \text{Big} & i=3 & j=3 \end{pmatrix}; \qquad Y_j \triangleq \begin{pmatrix} \text{Bad} & j=1 \\ \text{Not bad} & j=2 \\ \text{Neutral} & j=3 \\ \text{Good} & j=4 \\ \text{Very good} & j=5 \end{pmatrix}.$$

The universes of discourse of LVs are $U_{X1} = U_{X2} = U_{X3} = U_Y = R$.

Three types of membership functions are used in FLQM model: Gaussian membership function $\mu_{\mathrm{G}}(x) = e^{-\frac{1}{2}\left(\frac{x-\beta}{a}\right)^2}$; Bell membership function $\mu_{\mathrm{B}}(x) = \dfrac{1}{1+\left|\frac{x-\gamma}{a}\right|^{2\beta}}$ and Sigmoid membership function $\mu_{\mathrm{S}}(x) = \dfrac{1}{1+e^{-\alpha(x-\beta)}}$. The corresponding type of MF and the values of parameters are shown on Table 1.

For each input variable a degree of membership to the corresponding term is calculated.

2.2. Fuzzy inference. In the proposed model a Mamdani-type fuzzy inference (MFIS) system is chosen. As a result of MFIS a fuzzy output is obtained and this is the major reason for which MFIS are widely used in decision support applications [4, 5, 14]. There are four stages in the fuzzy inference process: (1) evaluation of the antecedent for each rule; (2) obtaining a conclusion for each rule; (3) aggregation of all conclusions and, (4) defuzzifying. The AND and THEN operators are implemented by min fuzzy T-norm, whereas the aggregation is im-

Table 1

Type and parameters of membership functions of terms

Terms	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}
MF	$\mu_S(x)$	$\mu_G(x)$	$\mu_G(x)$	$\mu_G(x)$	$\mu_S(x)$	$\mu_S(x)$	$\mu_G(x)$	$\mu_G(x)$	$\mu_G(x)$	$\mu_S(x)$
α	−20	0.05	0.08	0.05	20	−2	0.05	0.07	0.05	2
β	0	0	1.1	1.2	1.3	0	0.1	0.3	0.5	0.7
Terms	X_{31}	X_{32}	X_{33}		Y_1	Y_2	Y_3	Y_4	Y_5	
MF	$\mu_S(x)$	$\mu_B(x)$	$\mu_G(x)$		$\mu_G(x)$	$\mu_G(x)$	$\mu_G(x)$	$\mu_G(x)$	$\mu_G(x)$	
α	−0.3	20	0.3		0.1	0.1	0.1	0.1	0.1	
β	20	4	60		0	0.25	0.5	0.75	1	
γ		40								

plemented by max fuzzy T-conorm. For defuzzification of the output the centre of gravity method is used.

As there are three input variables with 5, 5 and 3 terms accordingly, the universe of all possible rules consists of 75 rules. In the proposed model 24 of the rules are chosen due to the expert opinion of the authors. Although these rules adequately describe the most important possible situations that might arise in the process of investment decision-making, the list of fuzzy rules can be extended without changing the system's architecture. The fuzzy rules model intuitively the decision making process and have IF–THEN form:

$$\text{IF } (r^* \text{ is } X_{1i}) \text{ AND } (s^* \text{ is } X_{2j}) \text{ AND } (q^* \text{ is } X_{3k}) \text{ THEN } (Q\text{-measure is } Y_p)$$
$$\text{for } i = 1, \ldots, 5;\ j = 1, \ldots, 5;\ k = 1, 2, 3 \text{ and } p = 1, \ldots, 5.$$

Some of the rules, implemented in FLQM model with their respective weights are shown on Table 2.

In the next stage the fuzzy rules are fired. At this point additional expert knowledge is taken into account by assigning weights to rule in the structure. In this way for a crisp input (r^*, s^*, q^*) the obtained membership values are:

$$\theta^* = \min\{\mu_i(r^*), \mu_i(s^*), \mu_i(q^*)\}$$

Table 2

Fuzzy rules for decision making

No	Return	Risk	q-ratio	Q-measure	Weight
1	Very high	Very low	Big	Very good	1
2	Very high	Low	Big	Very good	1
3	High	Very low	Big	Very good	1
4	High	Low	Big	Very good	1
5	Very high	Very low	Big	Good	0.8
…	…	…	…	…	…
24	Very low	High	Neutral	Not good	0.8

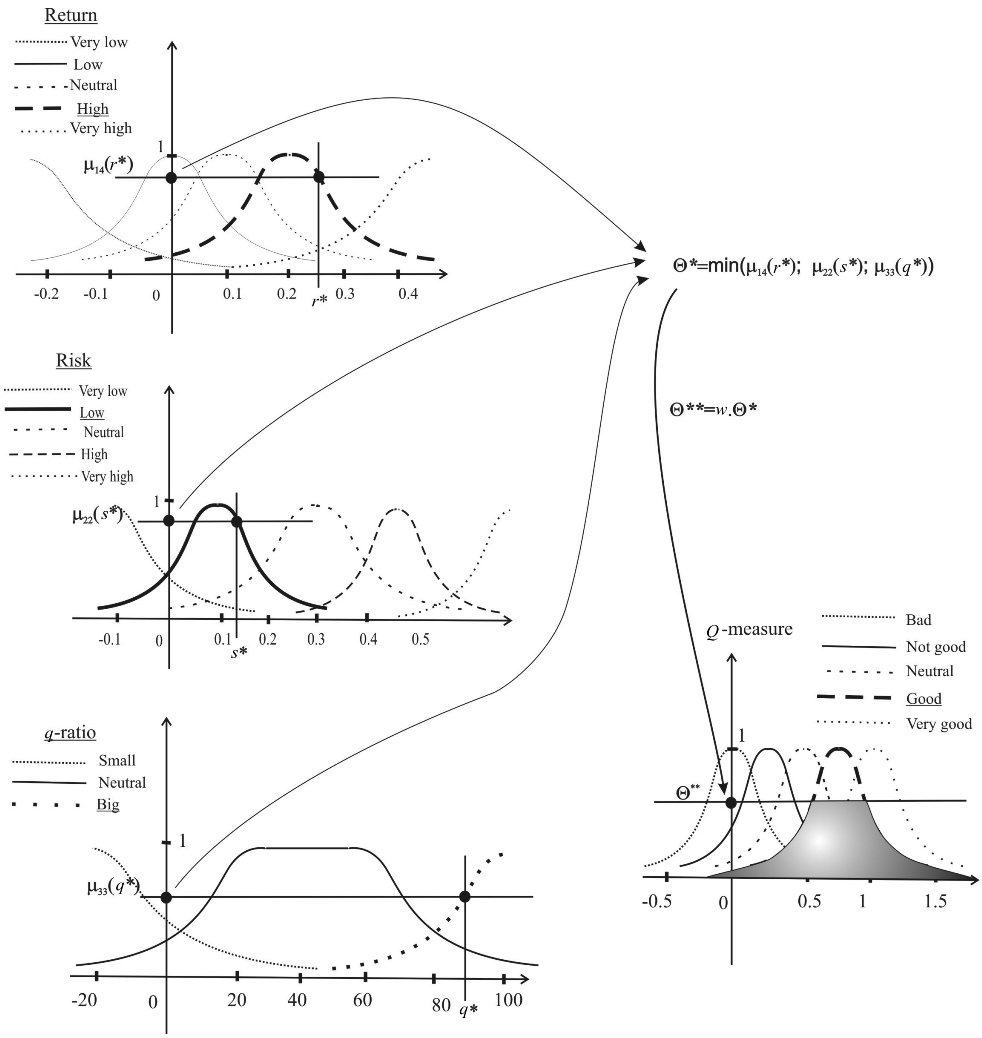

Fig. 1. Firing rule number 7: IF (Return is high) AND (Risk is low) AND (q-ratio is big) THEN (Q-measure is good)

and then respectively

$$\theta^{**} = w.\theta^{*},$$

where w is the corresponding weight of the rule. This procedure for the seventh rule in FLQM model is illustrated in Fig. 1.

After applying all the rules, several values for each term of the output variable Q-measure are calculated. Aggregation is the process of bringing together the outcomes of all the fuzzy rules [5]. Choosing a suitable aggregation operator is a key issue when a fuzzy system is designed. As an aggregation method the max fuzzy T-conorm is applied in the proposed model and thus the fuzzy output variable is obtained (Fig. 2).

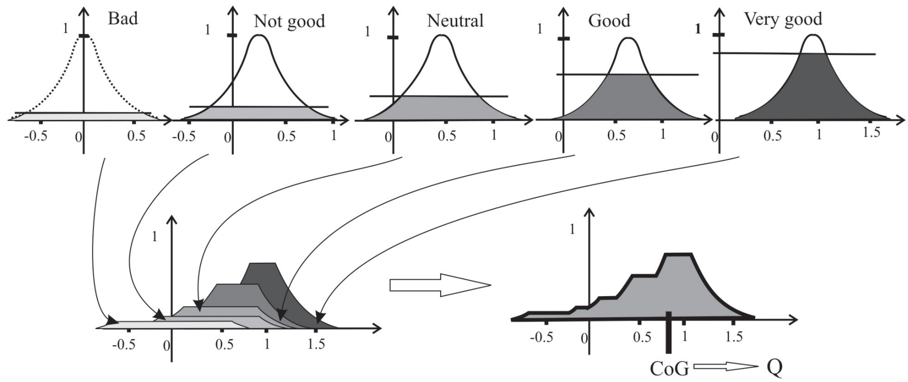

Fig. 2. Aggregation and defuzzification for obtaining the Q-measure of an asset

The overall fuzzy output generally constitutes a multimodal non-zero distribution of possible crisp values over a subset of the output space. In the defuzzification stage, one of those possible crisp values has to be selected. The design of a sound defuzzification method is important as it will affect the interpretation of the fuzzy response. A desirable defuzzification procedure should require a low computational effort to allow its implementation in real-time applications. At the same time, it should allow a smooth response and mapping accuracy to be obtained over all or most of the output space. What is more, a defuzzification method should ease the design of the fuzzy system and keep the decision making logic transparent to the user.

The centre of gravity (CoG) has been chosen as a defuzzification method in FLQM model:

$$\text{CoG}(Q) = \frac{\int_{-\infty}^{+\infty} xQ(x)\,dx}{\int_{-\infty}^{+\infty} Q(x)\,dx} \tag{1}$$

and thus a crisp value for the asset quality is obtained as an output of FLQM model.

3. Results. ***3.1. Software application.*** For testing the FLQM model a software application is created. It consists of three modules and a data base.

The first module is data managing module (DMM) and it is an application for collecting, storing and managing financial data in real time from the web page of Bulgarian Stock Exchange [8]. In addition, in this module calculations of precise measurements of important asset characteristics (return, risk and q-ratio) are implemented. The module consists of Requester, Parser, Filler and Calculator.

The second module is Q-measure fuzzy logic module which consists of applications based on fuzzy logic. Input data for this module are the crisp numerical values of asset characteristics from DMM. These crisp values are fuzzified and after applying the aggregation rules a fuzzy variable Q-measure for each of the assets is obtained. The output of this module is a defuzzified crisp value of Q-measure. For calculating the integrals in formula (1) the composite trapezoidal rule is implemented.

The third module is Portfolio construction module and this is where several portfolios are constructed. First, all assets in the database are sorted in descending order by their Q-measure. Then if the investor would hold a portfolio with no more than m assets, the first m assets are taken. Let $A = \{A_1, A_2, A_3, \ldots, A_m\}$ be the set of the top m financial assets. Now all possible combinations with $1, 2, \ldots, m$ elements are recorded. The number of these combinations is $2^m - 1$, because the empty set is not taken into consideration. Next, a portfolio is constructed for each combination of assets.

Let x_j be the share of asset A_j. Then according to [21] x_j is calculated as

$$x_j = \frac{Q_j}{\sum_{j=1}^{n} Q_j},$$

where n is the number of financial assets in the particular portfolio, $n = 1, 2, \ldots, m$, and Q_j is the Q-measure of A_j. Next, the portfolio return R_p, the portfolio risk σ_p and q-ratio of the portfolio q_p are calculated for all $2^m - 1$ portfolios according to the formulas:

$$R_p = \sum_{j=1}^{n} x_j r_j, \qquad \sigma_p = \sum_{j=1}^{n} x_j s_j, \qquad q_p = \frac{R_p}{\sigma_p}.$$

Finally, each portfolio is put through the Q-measure fuzzy logic module of the software application to obtain its Q-measure. The portfolios with their characteristics are stored in the database.

3.2. Individual financial asset management. In investment management the most important point is gaining high profit with lowest possible risk. However, for the non-speculative investor, it is essential to what extent these two characteristics (return and risk) are stable over time. The FLQM model is build on one additional characteristic: the q-ratio, which is the quotient of return and risk and reflects the degree to which the taken risk is justified by adequate returns. The conducted empirical tests for all assets listed on BSE for different periods of time show that the Q-measure is a proper indicator of the quality of the asset over time. If the Q-measure is less than 0.4 (whatever return and risk), a dramatic decrease of price occurs in up to about 3 months. At the same level and a Q-measure between 0.4 and 0.6 the price of the asset does not change significantly and even if it increases, the transaction costs will exceed the potential benefits. When the Q-measure is greater than 0.6, the asset price increases steadily and such an asset is considered suitable for purchase. Detailed results for individual asset management are published in [7].

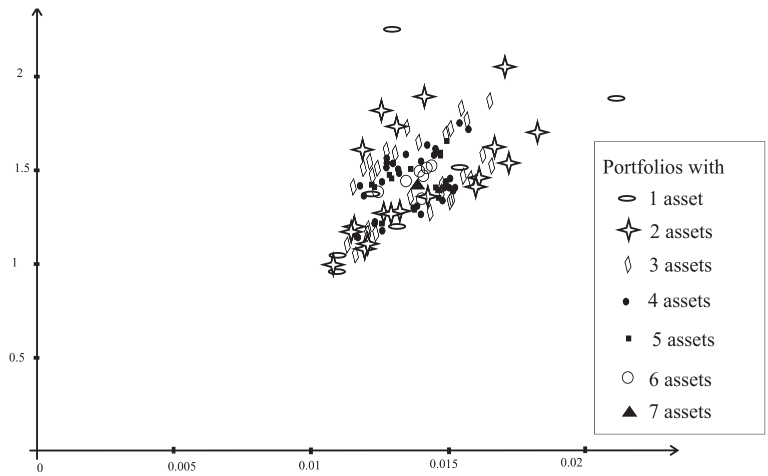

Fig. 3. 127 possible portfolios

3.3. Portfolio management. On 21.11.2012 the software system, built on FLQM model, detected seven assets from BSE with high Q-measures (above 0.8): 5BN, 6AS, 6A6, 5BD, 4EC, 3JR and 5ALB. If these assets are used, the portfolio constructing module creates 127 possible portfolios. The total investment capital is fixed to BGN 150 000 and naturally all the portfolios have Q-measure above 0.8. Each of these portfolios is represented as a point on the graph shown in Fig. 3, investment risk being plotted on the x axis, and the return being plotted on the y axis.

Investors can choose any portfolio depending on their preferences for return, risk or number of included shares.

4. Conclusion and future work. In this paper a Fuzzy Logic Q-measure model for assessing financial assets is proposed. This model is based on the Q-measure of an asset: a characteristic which combines return, risk and their ratio, and being modelled with fuzzy logic tools, it intuitively reflects the process of investment decisions in economic environment with enormous amount of data, which is often uncompleted and imprecise.

Major difference from existing models is that there are no requirements for probability distributions of returns as empirical tests on real data show absence of such distributions.

FLQM model has served as a base for creating a software system which is used for conducting tests on real data from BSE. Some of the results are presented in this paper. First, the system calculates the Q-measure of every asset on BSE and this quantity can be used in the process of investment decision. In addition, the system executes a procedure for portfolio allocation which allows the investors to base their decision on financial market information, provided by the model and on their personal preferences. One advantage is that as an output the investor can chose between several portfolios that differ in number of assets, return and risk but still show high Q-measures.

There are various directions for the future improvement of the model: adjusting the parameters of the membership function with a neural network; expanding the number of fuzzy rules and managing the investment (individual or portfolio) over time.

REFERENCES

[1] Angelova V. Investigations in the Area of Soft Computing. CIT, **9**, 2009, No 1, 18–24.

[2] Bojadziev G., M. Bojadziev. Fuzzy Logic for Business, Finance and Management, World Scientific, 1997.

[3] Campbell J., A. Lo, C. MacKinlay. The Econometrics of Financial Markets. Princeton University Press, 1997.

[4] Carlsson C., R. Fuller. Fuzzy Sets and Systems, 1996, No 78, 139–153.

[5] Fang Y. et al. Fuzzy Portfolio Optimization, Springer, 2008.

[6] Georgieva P. XVI Int. Symp. SIELA, vol. **1**, 2009, 101–106.

[7] Georgieva P., I. Popchev. Int. J. on Soft Computing, **3**, 2012, No 4, 21–38.

[8] Hanke J., A. Reitsch. Understanding Business Statistics, Homewood, 1990.

[9] Haugen R. Modern Investment Theory. NJ, Prentice-Hall, 2000.

[10] Ivanova Zl., Kr. Stoilova, T. Stoilov. Portfolio Optimization-Information Service in Internet. Sofia, Marin Drinov Academic Publishing House, 2005.

[11] Jang R. Fuzzy inference systems. NJ, Prentice-Hall, 1997.

[12] Judge G., R. Hill et al. Introduction to the Theory and Practice of Econometrics, New York, Wiley, 1988.

[13] Leland H. Financial Analysts J., **55**, 1999, No 1, 27–36.

[14] Mamdani E. H., S. Assilian. Int. J. of Man-Machine Studies, **7**, 1975, No 1, 1–13.

[15] Markowitz H. Portfolio Selection: Efficient Diversification of Investments. John Wiley and Sons, 1959.

[16] Nawrocki D. Financial Markets and Portfolio Management, 1992, No 2, 219–235.

[17] Peneva V., I. Popchev. Compt. rend. Acad. bulg. Sci., **61**, 2008, No 3, 293–300.

[18] Peneva V., I. Popchev. Compt. rend. Acad. bulg. Sci., **61**, 2008, No 5, 579–584.

[19] Peneva V., I. Popchev. Compt. rend. Acad. bulg. Sci., **62**, 2009, No 5, 551–558.

[20] Peneva V., I. Popchev. Compt. rend. Acad. bulg. Sci., **63**, 2010, No 7, 979–992.

[21] Popchev I., P. Georgieva. ITITEEE: 427–431, Springer Science, 2008.

[22] Zadeh L. Information Sciences, **8**, 1975, No 3, 199–249.

[23] Zadeh L., R. Bellman. Decision-making in a Fuzzy Environment. Manag. Science, 1970.

Burgas Free University
62, San Stefano Str.
8001 Burgas, Bulgaria
e-mail: `pgeorg@bfu.bg`

*Information and Communication Technologies
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 2
1113 Sofia, Bulgaria
e-mail: `ipopchev@iit.bas.bg`