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LOCALIZED FUNCTION METHOD APPLYING A SET OF SINE FUNCTIONS TO MODEL PHOTONIC CRYSTAL FIBRES

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Abstract

A development and an application of the localized function method based on the Galerkin method applying a set of sine functions to approximate the unknown mode fields of the localized modes propagating along the photonic crystal fibres (PCFs) is proposed. A way for considerably reducing the number of integrals in the case of symmetrical shapes of holes with respect to the axes of coordinate systems located at the centres of the holes (circular, elliptical, etc.) is also presented. The method does not require an expansion of the refractive index and thus inaccuracies of the expansion can be avoided. In the case of a circular form of the holes all integrals are solved analytically.

Key words: photonic crystal fibre, localized function method and Galerkin method, a set of sine functions

Introduction. Photonic crystal fibres [1] attract considerable attention since their waveguiding structure provides possibilities for creation of new or improved properties compared to the conventional optical fibres – endlessly single-mode operation; unique dispersion properties; very low or very high nonlinearity; low or high birefringence [2–5]. Thanks to such properties PCFs find applications in many fields such as: optical communications, sensor technology, fibre optic light sources, spectroscopy, biomedical studies [6–9], etc.

The high refractive index contrast of the materials requires full-vector methods [10] to model PCFs accurately. The most widely used methods are a plane wave expansion method (PWM) [11,12]; a localized function method (LFM) [13]; a beam propagation method (BMP) [14]; a finite-element method (FEM) [15]; a finite-difference method (FDM) in the time domain [16]; a finite-difference method (FDM) in the frequency domain [17] and a highly accurate semi-analytical multipole method [18]. A brief review of their merits and drawbacks is given in [17].

Here we offer a development and an application of the localized function method based on the Galerkin method applying a set of sine functions to approximate the unknown mode fields of the localized modes propagating along the PCF. Moreover, since one drawback of the method is the need for a calculation of many integrals [17], we propose a way to reduce considerably their number in the case of symmetrical holes shapes with respect to the axes of the coordinate systems located at the centres of the holes (circular, elliptical, etc.). The method does not require an expansion of the refractive index unlike PWM and LFM with Hermite–Gaussian functions, which is a major limiting factor for the accuracy of the calculations [10]. Due to this the regions with very small dimensions and a fine structure of interfaces can be considered exactly. In the case of a circular shape of the holes all integrals are solved analytically. We use some aspects of [19] applied to conventional optic fibres.

Formulation of the problem. We consider a translationally invariant PCF consisting of N_h holes located in an optical medium (host medium). Monochromatic light with angular frequency ω and time dependence $\exp(i\omega t)$ propagates along PCF in the direction of the axis z. We look for transverse distributions of the modal electric and magnetic fields with respect to a Cartesian coordinate system xOy with an origin at the lower left angle of a rectangular material domain with dimensions L_x $(0 \le x \le L_x)$ and L_y $(0 \le y \le L_y)$ comprising the cross-section of PCF with arbitrary locations of the holes. We assume that the electric and magnetic fields are zero at the domain boundaries. With an appropriate choice of the domain dimensions this approximation is reasonable due to the abrupt drop of the fields of the guided modes outside the core.

The modal electric and magnetic fields are solutions of the vector wave equations

$$\nabla^2\vec{E} + \nabla\left[\vec{E}\cdot\frac{\nabla n^2}{n^2}\right] + n^2k^2\vec{E} = 0, \quad \nabla^2\vec{H} + \left[\frac{\nabla n^2}{n^2}\times\left(\nabla\times\vec{H}\right)\right] + n^2k^2\vec{H} = 0,$$

where $\vec{E} \equiv \vec{E}(x,y,z)$ is the electric field vector, $\vec{H} \equiv \vec{H}(x,y,z)$ is the magnetic field vector, $k = \omega(\varepsilon_0\mu_0)^{1/2}$ is the wave number in free space, ε_0 is the dielectric permittivity of the vacuum, μ_0 is the magnetic permeability of the vacuum and $n \equiv n(x,y)$ is the refractive index of the medium.

In the Cartesian coordinate system the vector wave equations can be decomposed into x, y and z components. In this decomposition we use the fact that $\partial n/\partial z = 0$. The z-dependence of the components of the fields is assumed to be $\exp(i\beta z)$, where β is the longitudinal constant of propagation. It is enough to find solutions for the transverse components of the electric field $E_x(x,y)$, $E_y(x,y)$ and magnetic field $H_x(x,y)$, $H_y(x,y)$, because the longitudinal components $E_z(x,y)$ and $H_z(x,y)$ can be obtained from the transverse ones.

So, we look for solutions of the two systems of two coupled partial differential equations for the unknown x and y components of the electric field $E_x \equiv E_x(x,y)$, $E_y \equiv E_y(x,y)$ and the magnetic field $H_y \equiv H_y(x,y)$, $H_x \equiv H_x(x,y)$:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + (n^2k^2 - \beta^2)E_x + 2\frac{\partial}{\partial x}\left[E_x\frac{\partial \ln(n)}{\partial x} + E_y\frac{\partial \ln(n)}{\partial y}\right] = 0$$
$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + (n^2k^2 - \beta^2)E_y + 2\frac{\partial}{\partial y}\left[E_x\frac{\partial \ln(n)}{\partial x} + E_y\frac{\partial \ln(n)}{\partial y}\right] = 0,$$
$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (n^2k^2 - \beta^2)H_y - 2\frac{\partial \ln(n)}{\partial x}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = 0$$
$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + (n^2k^2 - \beta^2)H_x + 2\frac{\partial \ln(n)}{\partial y}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = 0.$$

The equation for H_x is placed after that for H_y because H_y and H_x have a transverse spatial distribution as E_x and E_y for plane waves in free space and also for guided modes.

We look for the solutions of these equations in the form

$$E_x(x,y) = \sum_{\mu=1}^{\infty}\sum_{\nu=1}^{\infty} A^E_{\mu\nu}\Phi_{\mu\nu}(x,y), \qquad E_y(x,y) = \sum_{\mu=1}^{\infty}\sum_{\nu=1}^{\infty} B^E_{\mu\nu}\Phi_{\mu\nu}(x,y),$$
$$H_y(x,y) = \sum_{\mu=1}^{\infty}\sum_{\nu=1}^{\infty} A^H_{\mu\nu}\Phi_{\mu\nu}(x,y), \qquad H_x(x,y) = \sum_{\mu=1}^{\infty}\sum_{\nu=1}^{\infty} B^H_{\mu\nu}\Phi_{\mu\nu}(x,y),$$

where $\Phi_{\mu\nu}(x,y) = [2/(L_xL_y)^{1/2}]\sin(\sigma_\mu x)\sin(\rho_\nu y)$ is a complete orthonormal set of sine functions which are orthogonal over the finite rectangular domain with dimensions L_x $(0 \le x \le L_x)$ and L_y $(0 \le y \le L_y)$:

$$\int_0^{L_x} dx \int_0^{L_y} dy\, \Phi_{\mu\nu}(x,y)\Phi_{\mu'\nu'}(x,y) = \delta_{\mu\mu'}\delta_{\nu\nu'}; \quad \sigma_\mu = \mu\pi/L_x; \quad \rho_\nu = \nu\pi/L_y;$$

μ, ν are integers; $A^E_{\mu\nu}$, $B^E_{\mu\nu}$, $A^H_{\mu\nu}$, $B^H_{\mu\nu}$ are unknown coefficients in the expansions of E_x, E_y, H_y and H_x, respectively.

Using Galerkin method the two systems of the two partial differential equations are converted into two systems each of $2m_xm_y$ coupled linear algebraic equations (m_x and m_y are the numbers of members in truncated sums) for the unknown coefficients $A^E_{\mu\nu}$, $B^E_{\mu\nu}$, $A^H_{\mu\nu}$, $B^H_{\mu\nu}$ (this procedure is well known; see for example [[19]]).

The double integrals over the material domain S of the studied quantities (f), obtained after using Galerkin method, can be presented as a sum of double integrals over the host medium and over the holes in it

$$\int_0^{L_x}\int_0^{L_y} f(x,y;n(x,y))\,dxdy = \sum_{i=1}^{N_h}\iint_{S_i} f(x,y;n_i)\,dxdy + \iint_{\text{host medium}} f(x,y;n_{\text{host}})\,dxdy.$$

In order not to integrate over the host medium we add and subtract integrals over the holes surfaces, in which the refractive indices are replaced by the refractive index of the host medium and obtain

$$\int_0^{L_x}\int_0^{L_y} f(x,y;n(x,y))\,dxdy = \int_0^{L_x}\int_0^{L_y} f(x,y;n_{\text{host}})\,dxdy + \sum_{i=1}^{N_h}\iint_{S_i} [f(x,y;n_i) - f(x,y;n_{\text{host}})]\,dxdy$$

e.g. the double integral over the domain with N_h interfaces is replaced by a sum of a double integral over a homogeneous medium with surface S and refractive index n_{host} (where the orthogonality of the sine functions can be used) and N_h homogeneous media with surfaces S_i, $i = 1,2,\ldots,N_h$ with changed refractive indices. Then the two systems of algebraic equations can be written as

$$(1)\qquad \sum_{\mu=1}^{m_x}\sum_{\nu=1}^{m_y}(M^E_{\mu'\nu',\mu\nu}A^E_{\mu\nu} + N^E_{\mu'\nu',\mu\nu}B^E_{\mu\nu}) = (\beta/k)^2 A^E_{\mu'\nu'}$$

$$(2)\qquad \sum_{\mu=1}^{m_x}\sum_{\nu=1}^{m_y}(R^E_{\mu'\nu',\mu\nu}A^E_{\mu\nu} + S^E_{\mu'\nu',\mu\nu}B^E_{\mu\nu}) = (\beta/k)^2 B^E_{\mu'\nu'}; \qquad \begin{array}{l}\mu' = 1,2,\ldots,m_x;\\ \nu' = 1,2,\ldots,m_y;\end{array}$$

$$(3)\qquad \sum_{\mu=1}^{m_x}\sum_{\nu=1}^{m_y}(M^H_{\mu'\nu',\mu\nu}A^H_{\mu\nu} + N^H_{\mu'\nu',\mu\nu}B^H_{\mu\nu}) = (\beta/k)^2 A^H_{\mu'\nu'}$$

$$(4)\qquad \sum_{\mu=1}^{m_x}\sum_{\nu=1}^{m_y}(R^H_{\mu'\nu',\mu\nu}A^H_{\mu\nu} + S^H_{\mu'\nu',\mu\nu}B^H_{\mu\nu}) = (\beta/k)^2 B^H_{\mu'\nu'}; \qquad \begin{array}{l}\mu' = 1,2,\ldots,m_x;\\ \nu' = 1,2,\ldots,m_y;\end{array}$$

where

$$(5)\qquad M^E_{\mu'\nu',\mu\nu} = \frac{4}{S}\sum_{i=1}^{N_h+1}\left[n^i_s I^i_{ssss} + 2\frac{\sigma_{\mu'}}{k^2}\ln(n^i_d)(\sigma_\mu I^i_{ccss} - \sigma_{\mu'} I^i_{ssss})\right];$$

$$(6)\qquad N^E_{\mu'\nu',\mu\nu} = \frac{8}{S}\sum_{i=1}^{N_h+1}\frac{\sigma_{\mu'}}{k^2}\ln(n^i_d)(\rho_\nu I^i_{sccs} + \rho_{\nu'} I^i_{scsc});$$

$$(7)\qquad R^E_{\mu'\nu',\mu\nu} = \frac{8}{S}\sum_{i=1}^{N_h+1}\frac{\rho_{\nu'}}{k^2}\ln(n^i_d)(\sigma_\mu I^i_{cssc} + \sigma_{\mu'} I^i_{scsc})$$

$$(8)\quad S^{E}_{\mu'\nu',\mu\nu} = \frac{4}{S}\sum_{i=1}^{N_h+1}\left[n_s^i I^i_{ssss} + 2\frac{\rho_{\nu'}}{k^2}\ln(n_d^i)(\rho_\nu I^i_{sscc} - \rho_{\nu'} I^i_{ssss})\right];$$

$$(9)\quad M^{H}_{\mu'\nu',\mu\nu} = \frac{4}{S}\sum_{i=1}^{N_h+1}\left[n_s^i I^i_{ssss} + 2\frac{\sigma_{\mu}}{k^2}\ln(n_d^i)(\sigma_{\mu'} I^i_{ccss} - \sigma_{\mu} I^i_{ssss})\right];$$

$$(10)\quad N^{H}_{\mu'\nu',\mu\nu} = -\frac{8}{S}\sum_{i=1}^{N_h+1}\frac{\rho_\nu}{k^2}\ln(n_d^i)(\sigma_\mu I^i_{cscs} + \sigma_{\mu'} I^i_{sccs});$$

$$(11)\quad R^{H}_{\mu'\nu',\mu\nu} = -\frac{8}{S}\sum_{i=1}^{N_h+1}\frac{\sigma_\mu}{k^2}\ln(n_d^i)(\rho_\nu I^i_{cscs} + \rho_{\nu'} I^i_{cssc});$$

$$(12)\quad S^{H}_{\mu'\nu',\mu\nu} = \frac{4}{S}\sum_{i=1}^{N_h+1}\left[n_s^i I^i_{ssss} + 2\frac{\rho_{\nu}}{k^2}\ln(n_d^i)(\rho_{\nu'} I^i_{sscc} - \rho_{\nu} I^i_{ssss})\right];$$

$$n_s^i = \begin{cases} n_i^2 - n^2_{\text{host}} & i = 1,2,\dots,N_h \\ n^2_{\text{host}} - n^2_{\mu\nu}, & i = N_h+1 \end{cases};\quad n_d^i = \begin{cases} n_i/n_{\text{host}} & i = 1,2,\dots,N_h \\ n_{\text{host}} & i = N_h+1 \end{cases};$$

n_i is the constant refractive index of the i-th hole with the surface S_i, n_{host} is the constant refractive index of the host medium, $n^2_{\mu\nu} \equiv (\sigma_\mu^2 + \rho_\nu^2)/k^2$ is a dimensionless quantity, the number $N_h + 1$ is referred to the material domain,

$$I^i_{ssss} = \begin{cases} I^{S_i}_{ssss} = \iint_{S_i} \sin(\sigma_\mu x)\sin(\sigma_{\mu'} x)\sin(\rho_\nu y)\sin(\rho_{\nu'} y)\,dxdy, & i = 1,2,\dots,N_h \\ I^{S}_{ssss} = \iint_{S} \sin(\sigma_\mu x)\sin(\sigma_{\mu'} x)\sin(\rho_\nu y)\sin(\rho_{\nu'} y)\,dxdy, & i = N_h+1, \end{cases}$$

$$I^i_{ccss} = \begin{cases} I^{S_i}_{ccss} = \iint_{S_i} \cos(\sigma_\mu x)\cos(\sigma_{\mu'} x)\sin(\rho_\nu y)\sin(\rho_{\nu'} y)\,dxdy, & i = 1,2,\dots,N_h \\ I^{S}_{ccss} = \iint_{S} \cos(\sigma_\mu x)\cos(\sigma_{\mu'} x)\sin(\rho_\nu y)\sin(\rho_{\nu'} y)\,dxdy, & i = N_h+1. \end{cases}$$

The definitions of the remaining integrals are analogous.

Let us consider the i-th hole. A local coordinate system $x'O_iy'$ with an origin located at the centre of the i-th hole and axes parallel to the axes of the global coordinate system xOy is introduced (Fig. 1). We consider the case when the hole form is symmetrical with respect to the local axes. After variable changing the integral $I^{S_i}_{ssss}$ can be written in the local coordinate system as

$$I^{S_i}_{ssss} = \int_{-a_i}^{a_i} dx' \sin[\sigma_\mu(x_i + x')]\sin[\sigma_{\mu'}(x_i + x')] \times \int_{-\varphi_i(x')}^{\varphi_i(x')} dy' \sin[\rho_\nu(y_i + y')]\sin[\rho_{\nu'}(y_i + y')]$$

$$= \left\{ \iint dx'dy' \cos[\sigma_m(x_i + x')]\cos[\rho_m(y_i + y')] \right.$$

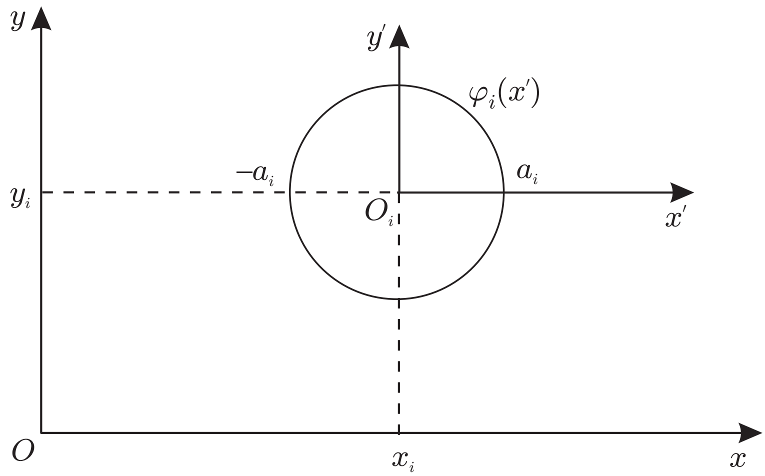

Fig. 1. A local coordinate system $x'O_iy'$ with an origin located at the centre of the i-th hole of a PCF and axes parallel to the axes of the global coordinate system xOy

$$- \iint dx'dy' \cos[\sigma_m(x_i + x')] \cos[\rho_p(y_i + y')]$$
$$- \iint dx'dy' \cos[\sigma_p(x_i + x')] \cos[\rho_m(y_i + y')]$$
$$+ \iint dx'dy' \cos[\sigma_p(x_i + x')] \cos[\rho_p(y_i + y')] \Big\}/4,$$

where x_i and y_i are the coordinates of the centre of the i-th hole in the global coordinate system; $-a_i$ and a_i are the lower and upper limits of changing of the local variable x'; $-\varphi_i(x')$, $\varphi_i(x')$ are the lower and upper limits of changing of the local variable y' (the integration limits are dropped for brevity in the last expression); $\sigma_m \equiv \sigma_\mu - \sigma_{\mu'}$; $\sigma_p \equiv \sigma_\mu + \sigma_{\mu'}$; $\rho_m \equiv \rho_\nu - \rho_{\nu'}$; $\rho_p \equiv \rho_\nu + \rho_{\nu'}$.

The global variables x_i and y_i can be separated from the local variables x' and y' using the formulae for adding of trigonometric functions and the fact that the integral over odd function is equal to zero. Then

$$\iint dx'dy' \cos[\sigma_j(x_i + x')] \cos[\rho_k(y_i + y')]$$
$$= \cos(\sigma_j x_i) \cos(\rho_k y_i) \iint dx'dy' \cos(\sigma_j x') \cos(\rho_k y'), \quad j, k = m, p.$$

This presentation makes it possible to reduce the number of integrals to the four integrals I_{mm}, I_{mp}, I_{pm}, I_{pp}:

$$I^{S_i}_{ssss} = (c_{mm}I_{mm} - c_{mp}I_{mp} - c_{pm}I_{pm} + c_{pp}I_{pp})/4;$$

$$I^{S_i}_{ccss} = (c_{mm}I_{mm} - c_{mp}I_{mp} + c_{pm}I_{pm} - c_{pp}I_{pp})/4;$$
$$I^{S_i}_{sscc} = (c_{mm}I_{mm} + c_{mp}I_{mp} - c_{pm}I_{pm} - c_{pp}I_{pp})/4;$$
$$I^{S_i}_{sccs} = (-s_{mm}I_{mm} + s_{mp}I_{mp} - s_{pm}I_{pm} + s_{pp}I_{pp})/4;$$
$$I^{S_i}_{scsc} = (s_{mm}I_{mm} + s_{mp}I_{mp} + s_{pm}I_{pm} + s_{pp}I_{pp})/4;$$
$$I^{S_i}_{cssc} = (-s_{mm}I_{mm} - s_{mp}I_{mp} + s_{pm}I_{pm} + s_{pp}I_{pp})/4;$$
$$I^{S_i}_{cscs} = (s_{mm}I_{mm} - s_{mp}I_{mp} - s_{pm}I_{pm} + s_{pp}I_{pp})/4,$$

where $c_{jk} = \cos(\sigma_j x_i)\cos(\rho_k y_i)$, $s_{jk} = \sin(\sigma_j x_i)\sin(\rho_k y_i)$ are functions depending only on the location of holes with respect to the global coordinate system;

$$I_{jk} = \int_{-a_i}^{a_i}\int_{-\varphi_i(x')}^{\varphi_i(x')} dx'dy'\cos(\sigma_j x')\cos(\rho_k y'), \quad j,k = m,p$$

are integrals which depend on the hole shape, but not on its location. For all holes with identical shapes the four integrals can be solved only once. This reduces considerably the integrals numbers and the calculation time especially when the integrals are numerically calculated. These four integrals can be solved analytically when the hole cross section has a circular form. In this case

$$\begin{aligned} I^{s_i}_{ssss} = \int_{x_i-a_i}^{x_i+a_i} dx\sin(\sigma_\mu x)\sin(\sigma_{\mu'} x)\int_{y_i-\sqrt{a_i^2-(x-x_i)^2}}^{y_i+\sqrt{a_i^2-(x-x_i)^2}} dy\sin(\rho_\nu y)\sin(\rho_{\nu'} y) \\ = \int_{-a_i}^{a_i} dx'\sin[\sigma_\mu(x_i+x')]\sin[\sigma_{\mu'}(x_i+x')] \\ \times\int_{-\sqrt{a_i^2-(x')^2}}^{\sqrt{a_i^2-(x')^2}} dy'\sin[\rho_\nu(y_i+y')]\sin[\rho_{\nu'}(y_i+y')] \\ = (c_{mm}I_{mm} - c_{mp}I_{mp} - c_{pm}I_{pm} + c_{pp}I_{pp})/4, \end{aligned}$$

where

$$I_{jk} = \int_{-a_i}^{a_i}\int_{-\sqrt{a_i^2-(x')^2}}^{\sqrt{a_i^2-(x')^2}} dx'dy'\cos(\sigma_j x')\cos(\rho_k y'), \quad j,k = m,p.$$

In order to solve analytically the double integral the following identity is used:

$$\int_{-a}^{a} dx\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\cos(Ax+By) = \frac{2\pi a}{\sqrt{A^2+B^2}}J_1\left(a\sqrt{A^2+B^2}\right),$$

where the used here coordinates x, y and parameters A, B and a are related only to this identity and are not connected with the global and local variables and parameters used so far. J_1 is the Bessel function of order 1 [20]. The integral $\iint dx'dy' \sin(\sigma_j x')\sin(\rho_k y')$ is subtracted from I_{jk} and we obtain

$$I_{jk} = \int_{-a_i}^{a_i} \int_{-\sqrt{a_i^2-(x')^2}}^{\sqrt{a_i^2-(x')^2}} dx'dy' \cos(\sigma_j x' + \rho_k y') = \frac{2\pi a_i}{\sqrt{\sigma_j^2+\rho_k^2}} J_1(a_i\sqrt{\sigma_j^2+\rho_k^2}),$$

$j,k = m,p$. So, in the case of circular holes all integrals are solved analytically.

Systems of algebraic equations (1), (2) and (3), (4) can be written in the form of matrix eigenvalue equations: $\hat{C}^E\vec{X}^E = (\beta/k)^2\vec{X}^E$; $\hat{C}^H\vec{X}^H = (\beta/k)^2\vec{X}^H$, where

$$\hat{C}^E \equiv \begin{pmatrix} \hat{M}^E & \hat{N}^E \\ \hat{R}^E & \hat{S}^E \end{pmatrix}; \hat{C}^H \equiv \begin{pmatrix} \hat{M}^H & \hat{N}^H \\ \hat{R}^H & \hat{S}^H \end{pmatrix}; \hat{M}^E, \hat{N}^E, \hat{R}^E, \hat{S}^E, \hat{M}^H, \hat{N}^H, \hat{R}^H, \hat{S}^H$$

are matrices consisting of the coefficients $M^E_{\mu'\nu',\mu\nu}$; $N^E_{\mu'\nu',\mu\nu}$; $R^E_{\mu'\nu',\mu\nu}$; $S^E_{\mu'\nu',\mu\nu}$; $M^H_{\mu'\nu',\mu\nu}$; $N^H_{\mu'\nu',\mu\nu}$; $R^H_{\mu'\nu',\mu\nu}$; $S^H_{\mu'\nu',\mu\nu}$, respectively; $\vec{X}^E = (\vec{A}^E, \vec{B}^E)^T$ and $\vec{X}^H = (\vec{A}^H, \vec{B}^H)^T$ are eigenvectors, consisting of coefficients $A^E_{\mu\nu}$; $B^E_{\mu\nu}$; $A^H_{\mu\nu}$; $B^H_{\mu\nu}$ and $(\beta/k)^2$ are unknown eigenvalues.

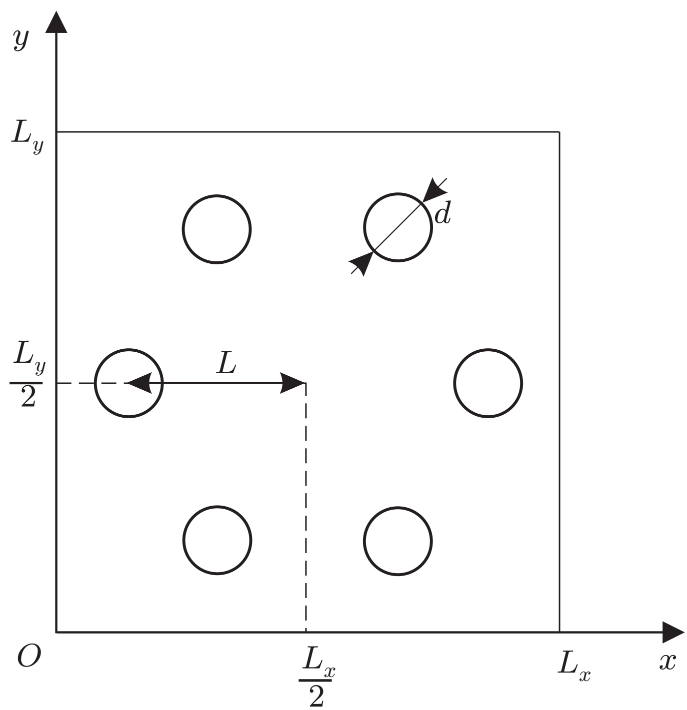

Fig. 2. Cross-section of a PCF consisting of 6 identical cylindrical air holes with a diameter $d = 1.0$ µm arranged in a hexagonal lattice with a constant (a pitch) $\Lambda = 2.3$ µm

The above derivation is incorporated into a single Visual Fortran 6.5 code. It calculates matrices elements, modal effective indices and transverse components of both the electric and magnetic field propagating along the PCF. EISPACK [21] is incorporated into the Fortran programme and is used to solve the eigenvalue equations.

Numerical results. The PCF example, calculated by the multipole method, is used in this paper since here analytical results are available for comparison. The PCF consists of 6 cylindrical air holes each with a diameter 1.0 µm and refractive index $n_i = 1.0$ $(i = 1, 2, \ldots, 6)$ arranged in a hexagonal lattice with a constant (a pitch) $\Lambda = 2.3$ µm within a host medium with refractive index $n_{\text{host}} = 1.44390356$ (Fig. 2).

According to the multipole method [22], the accurate effective index of the fundamental mode at wavelength $\lambda = 1.56$ µm would be $1.42078454 + i7.20952 \times 10^{-4}$.

Here we are solving the eigenvalue equation for a transverse magnetic field which is continuous at the interfaces unlike the electric field and requires less sine functions to be approximated accurately. In order to find the solution the numbers

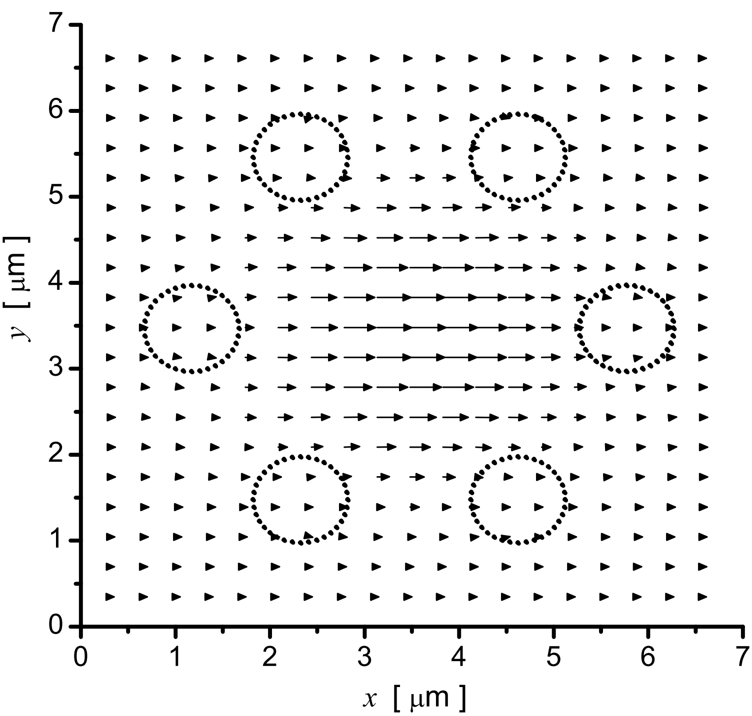

Fig. 3. A distribution of the transverse magnetic field $\vec{H}_t(x, y)$ of the fundamental mode over the cross section of the PCF. The effective index is $n_{\text{eff}} = 1.420790647751640$, $m_x = m_y = 68$ are expansion terms, $L_x = L_y = 6.96$ µm are dimensions of the material domain

of expansion terms and the dimensions of the material domain are changed from 5 to 71 and from $2\Lambda + 0.2$ µm to 4Λ, respectively. The essential part of the results is shown on Table 1. It can be seen from Table 1 that the value of n_{eff} with smallest relation errors $\Delta_m = -9.520 \times 10^{-7}$ and $\Delta_L = 2.400 \times 10^{-8}$ is a solution of the problem, $n_{\text{eff}} = 1.420790647751640$. The radiation loss is neglected. The relative error between the solution obtained by a multipole method and this solution is -4.3×10^{-6}.

The vector distribution of the transverse magnetic field $\vec{H}_t(x, y) = \vec{H}_x(x, y) + \vec{H}_y(x, y)$ over the PCF cross section is given in Fig. 3.

Conclusions. We have presented a development and an application of the localized function method based on Galerkin method applying a set of sine functions for numerical modelling of PCFs with arbitrary locations of holes without expanding of the refractive index. The above derivation is realized as a single Visual Fortran 6.5 programme. It calculates matrix elements, modal effective indices and transverse components of both the electric and magnetic fields propagating along the PCF. EISPACK [21] is incorporated in the Fortran programme and is used to solve the eigenvalue equations. Numerical results obtained by the programme are presented for the case of circular holes. The number of the integrals is reduced considerably in the case of symmetric holes shapes. For every element of matrices $\hat{M}$, $\hat{N}$, $\hat{R}$, $\hat{S}$ for both the electric and magnetic fields $7N_h$ integrals are solved for the holes. In the case of identical symmetrical shapes their

Table 1

Values of the effective index for the fundamental mode. Here n_{eff} are solutions of the effective index for which the smallest relative error Δ_m between two successive solutions with different terms in their expansions is obtained at dimensions L_x and L_y. The values of m_x and m_y are the expansion terms necessary to achieve Δ_m. It is assumed that $m_x = m_y$ and $L_x = L_y$. Δ_L are the relative errors for two solutions at two successive values of the dimensions of the material domain

$L_x = L_y$ [µm]	$m_x = m_y$	n_{eff}	Δ_m	Δ_L
6.80	70	1.420719731100830	-1.410×10^{-6}	–
6.82	70	1.420728725549000	-1.410×10^{-6}	6.330×10^{-6}
6.84	68	1.420737677303520	$+1.010 \times 10^{-7}$	6.300×10^{-6}
6.86	64	1.420747668278230	$+1.800 \times 10^{-7}$	7.032×10^{-6}
6.88	70	1.420755232661020	-1.421×10^{-6}	5.324×10^{-6}
6.90	70	1.420763935483380	-8.610×10^{-6}	6.120×10^{-6}
6.92	66	1.420775176219890	-3.780×10^{-7}	7.911×10^{-6}
6.94	60	1.420790612680160	-2.490×10^{-6}	10.865×10^{-6}
6.96	68	1.420790647751640	-9.520×10^{-7}	2.400×10^{-8}
6.98	70	1.420798387400970	-9.940×10^{-7}	5.446×10^{-6}

number is 4. In the test example the number of terms in the expansion of the magnetic field over sine functions is relatively small, about 70 terms along each of the axes. The considered model will be applied to calculating of PCFs with realistic configurations.

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